

Static Analysis of Railway Overheads Considering Pantograph Effect

Abdullah Demir*¹

¹Civil Engineering Department, Middle East Technical University; abdemir@metu.edu.tr

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Abstract:

This article is concerned with the static analysis of structural cables used in railway overheads. Structural analysis computer program named ANSYS is used for analysis. Two effects are considered in the analysis. First one is the bending behavior effect of cables. BEAM188 in addition to LINK10 and LINK180 is used to see the difference in case of additional bending effect. Besides, LINK10 and LINK180 is also compared. Second one is the effect of pantograph. Pantograph is modelled as a contact element instead of a force. Accordingly, some sample cable systems similar to railway overhead are analyzed.

Keywords: *Railway overheads, structural cable, sliding contacts, nonlinear analysis*

* Corresponding Author

1. Introduction

Overhead catenary system is the transmission of energy part of the railway systems. As being on move, there should be a contact between train and overhead system. This contact is provided by pantographs. Pantograph adjusts the contact between train and overhead system. An overhead system consists of many parts: cables, clamps, registration arms etc. (Brown, 1927). Basically, three different type of cables form a cable net having special name; overhead catenary, see Figure 1 (Lopez-Garcia et al., 2006). These cables are messenger, dropper and contact cables. The main issue in overhead system design is to keep the contact cable parallel to the railway. However, it is not easy due to sag of catenaries. Although engineers use messenger cable to minimize the sag of contact cable, there have been conflicts on cable net forms; mainly depending on dropper lengths. There are many researches about overhead catenaries. Most of them are related with dynamic properties of overhead systems (Wu and Brennan, 1998, Nordstrøm Jensen and True, 1998, Wu and Brennan, 1999, Arnold and Simeon, 2000, Cho, 2008, Cho et al., 2010). Besides these dynamic analysis researches, some researchers tried to find an optimal initial shape playing by dropper lengths (Lopez-Garcia et al., 2006, Tur et al., 2014). This optimal shape finding problems are named as initial equilibrium problems. There are several methods to tackle initial equilibrium problems; force density method and nonlinear displacement method (Dischinger, 1953). One more method it proposed by Lopez-Garcia, Carnicero and Torres (Lopez-Garcia et al., 2006). Some researchers use line elasticity (Poole and Huckins III, 1972) approach to analyze the system due to complexity of the structure. Line elasticity of contact cable should be kept uniform through-

out its geometry to minimize stiffness irregularities. This condition can be satisfied by playing with the shape of system which bases on dropper lengths. Satisfying this condition will optimize the cable structure (Galeotti and Toni, 1997). Although researches have been focused on dynamic analysis and optimization of overhead catenaries, this paper is concerning about static analysis of it. There are many static solution methods for cable, proposed by researchers. Those methods can be categorized into two: Closed-Form solutions and Iterative FEM solutions. Closed-Form solution methods were first proposed by Dischinger F. in 1949 (Dischinger, 1953). Detailed formulations can be found in Megson's book published in 2005 (Megson, 2005). Iterative FEM solutions were first proposed by Michalos J. and Brinstiel C. in 1962 (Michalos and Brinstiel, 1960). The method (Michalos and Brinstiel, 1960) is named as method of imaginary reactions. Many researchers including Demir A. (Demir, 2011) have been using this method now. Thesis of Demir A. (Demir, 2011) is recommended to readers interested in static analysis methods of cables. Instead of using those methods a commercial structural analysis program named as ANSYS will be used in this research as used in many researches dealing with the static analysis of cables (Zhang and Liu, 2005, Pakrastinsh et al., 2006, Finotto et al., 2013, Chen et al., 2014, Gesualdo et al., 2014).

2. Building up the Model

As briefed above, there are many methods about solution of cable statically. Iterative FEM solutions are accepted ones. Iterative FEM solutions can also be categorized into two according to initial condition assumptions: assuming initial shape or initial reaction. Researchers define an initial state for solution by assuming either of them.

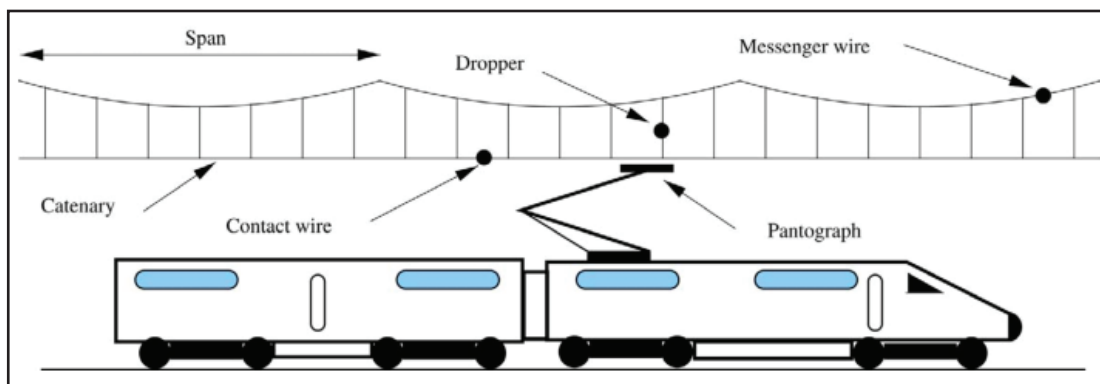


Figure 1: A sample train with pantograph and overhead catenary system (Lopez-Garcia et al., 2006).

In method of imaginary reactions, an initial reaction is needed to start the solution procedure. In contrast, in classical FEM solutions, initial shape is needed to start the solution procedure as in ANSYS. Although, method of imaginary reactions does not interested in the initial geometry of the cable, length of the cable should be known in either iterative solution method. Therefore, if static solution of cable is desired, cable length has to be known. This is a priority because analysis of cable strictly depends on the cable length which could be different than the span length. If a cable model having shorter/longer length than its span length is needed to be analyzed by a classical FEM, a displacement has to be applied one of the support which is initially placed at a different position due to cable length condition. It was seen that cable length in overhead catenary system did not mentioned in previous researches. Instead of cable length, cable tensions were mentioned and designs and optimizations had been done based on tensions. However, as explained above, a classical FEM solution needs the initial geometry of the cable to determine the tensions. Nevertheless, initial strain can be applied, which will change the initial length of the structure, to satisfy the tension condition. Some finite elements in ANSYS have that option like LINK10. Another issue about static cable analysis to be briefed is the convergence of the solution. As known, cable has very high nonlinearity in which geometric nonlinearity is concerned in this paper. Therefore, it is very hard to solve a cable statically by a classical FEM method. Engineers/researchers can face with non-converged solutions or even with converged solutions different from each other. The reason of first situation can be overcome by playing with the solution criterions or applied constraints. The second situation, more ingenious one, can be witnessed by an engineer point of view. Eventually, it should be kept in mind that it is hard to converge a static analysis of a cable structure by classical FEM method even with ANSYS.

2.1. Finite Elements

ANSYS presents many finite elements to model a structure. Some of them fit it, some not. Some finite elements can be used to model different structures by their special options. For example PIPE59 is a finite element written to model offshore cables which are immersed to the wa-

ter. However, other pipe elements or cable element can be used to model an offshore cable by a well definition. Before in ANSYS 13, there were two finite elements named LINK10 and LINK180 to model a cable structure. Although LINK10 is removed from the interface of the new version of ANSYS 14, LINK10 can be used with log file. There are some differences between these elements. First difference is; LINK10 has an initial strain option but LINK180 does not. As mentioned before, this property can be needed for different type of structures; especially for models need to define the tension of the elements like overhead catenary systems. Second one is; LINK10 has an option which defines additional stiffness to itself to increase the possibility of convergence, in contrast LINK180 does not. Although this additional stiffness change the results, it could be vital to find a solution.

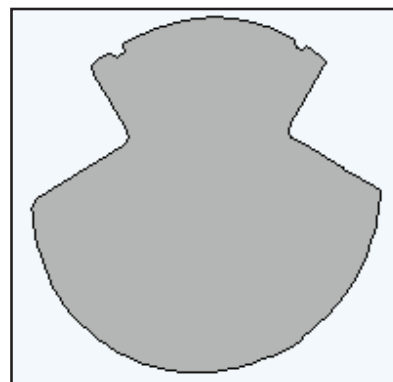


Figure 2: Contact wire cross-section

Inertia effects is not considered in both finite elements; LINK10 and LINK180. Although cable can be assumed as having zero bending moment capacity, if more accurate results are needed or the cross-sectional area of cable is larger in comparison with its length, it would be better to take the bending stiffness of the cable into account.

In this case, BEAM188 element can be used to model the cable. Although it is suggested to use BEAM188 element for cables having larger cross-sections, it should be noted that; cables having tangential geometry does not have the same bending moment capacity with a bar having same length and cross-sectional area. However, the contact wires used in overhead catenary systems do not have tangential geometry, see Figure 2. Therefore, BEAM188 is a convenient finite element to take the bending stiffness effect into account correctly.

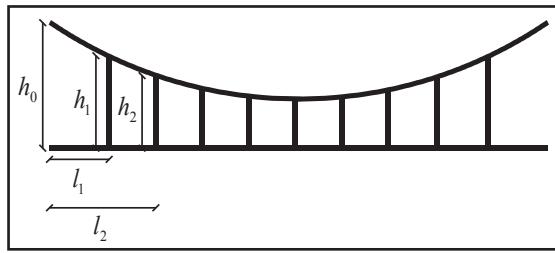


Figure 2: Contact wire cross-section

2.2. Sample Model 1

First model to be introduced is the simplest one. Three researches (Arnold and Simeon, 2000, Lopez-Garcia et al., 2006, Tur et al., 2014) have been done using this simple model. These researches deal with the dropper lengths of the model in which there are two dropper having same length due to symmetry. Material and geometric properties of cables used in model is summarized in Table 1. The shape properties of it are in Table 2 and dropper lengths and longitudinal positions found in each research are shown in Table 3. These shape properties can be seen in the sample sketch in Figure 3.

Table 1: Cable properties of model 1

Property Name	Contact Cable	Messenger Cable	Droppers
Mass per Unit Length (kg/m)	1.068	0.6	0.14
Clamp Mass (kg)	0.25	0.25	
Tension (kN)	15	15	
Elastic Modulus (GPa)	120	200	200
Cross-Sectional Area (mm ²)	120	76.43	17.83
Specific Mass (kg/m ³)	8930	7850	7850

Table 2: Shape properties of model 1

Property Name	Value
Span length (m)	20
Max. distance btw contact & messenger h_0 (m)	1
Number of droppers	2

Table 3: Dropper lengths and positions of model 1

Droppers	Longitudinal position of droppers l (m)	Dropper lengths h (m)		
		Arnold & Simeon (Arnold and Simeon, 2000)	Lopez-Garcia et al (Lopez-Garcia et al., 2008)	Tur et al (Dischinger, 1953)
1	5.5	0.95	0.9579	0.954
2	14.5	0.95	0.9579	0.954

Model is analyzed for three different dropper length cases with two different elements: LINK10 and LINK180. Being symmetric structure cable tensions at both ends are same, so resultant reaction at one end of the cables are compared for both messenger and contact cables.

It can be seen in Table 4 that solutions of LINK10 and LINK180 ends up with almost same results, so LINK10 will be used in further sample models. Other comparison can be done between researches: Although researchers found dropper lengths having 0.8% max difference, reaction differences reaches to 8% in contact cable and 1% in messenger cable as can be seen on Table 4.

Table 4: Results of model 1 with LINK10 and LINK180

Researcher name	Reaction at contact cable (N)		Reaction at messenger cable (N)	
	LINK10	LINK180	LINK10	LINK180
Arnold & Simeon (Arnold and Simeon, 2000)	1585.659	1585.759	3211.57	3211.37
Lopez-Garcia et al (Lopez-Garcia et al., 2008)	1710.614	1710.614	3152.375	3152.175
Tur et al (Dischinger, 1953)	1648.286	1648.286	3183.222	3183.122

Results seen in Table 4 are for models having no bending stiffness due to be modelled by finite elements LINK10 and LINK180. Researchers (Arnold and Simeon, 2000, Lopez-Garcia et al., 2006, Tur et al., 2014) defined a bending stiffness for contact cable, so it is desired to see the bending stiffness effect. In that sense, only contact cable is modelled by BEAM188 (others are modelled by LINK10) having circular cross-section instead of its real section seen in Figure 2. Table 5 shows the results of the model built up with LINK10 & BEAM188. It is seen that reactional difference decreases to 3% from 8%. Another result is that: If bending effect is taken into account, tension in contact cable will increase and tension in messenger cable will decrease.

Table 5: Results of model 1 with LINK10 & BEAM188

Researcher name	Reaction at contact cable (N)	Reaction at messenger cable (N)
Arnold & Simeon (Arnold and Simeon, 2000)	2155.396	2988.785
Lopez-Garcia et al (Lopez-Garcia et al., 2008)	2233.099	2939.303
Tur et al (Dischinger, 1953)	2193.962	2965.243

Researchers (Arnold and Simeon, 2000, Lopez-Garcia et al., 2006, Tur et al., 2014) defined an initial tension in cables, however results shown on Table 4 and Table 5 have been achieved in constant length condition with no defined initial stress. Initial tension condition can be satisfied by applying initial strain to the element LINK10. If there are other finite elements like BEAM188, initial tension condition can be satisfied by applying thermal changes. Applying either case will end up with same result for same models.

Comparing the researches with each other is not the aim of this paper, however one can see that all models will give same result if initial tension condition of them is satisfied. Nevertheless, it is better to note that; tension conditions cannot be satisfied with applying same constraints (initial strain or thermal change), which leads to different cable lengths. So, it can be concluded like that; researchers satisfied the same conditions with different cable lengths.

2.3. Sample Model 2

Second model is more complex one which reflects a real overhead catenary. Two researches (Wu and Brennan, 1998, Nordstrøm Jensen and True, 1998) have been done using this model.

Inputs of model can be seen in Table 6, 7, and 8. In previous model, comparison between models built by LINK10 and LINK180 has been done and it is seen that; there is almost no difference between them. Besides bending stiffness effect of contact cable is witnessed. In the second model, it is aimed to show the bending stiffness effect of messenger cable.

Table 6: Cable properties of model 2

Property Name	Contact Cable	Messenger Cable	Droppers
Mass per Unit Length (kg/m)	0.987	0.605	0
Clamp Mass (kg)	0.2	0.2	
Tension (kN)	12	12	
Elastic Modulus (GPa)	120	200	200
Cross-Sectional Area (mm ²)	110	77	17.83
Specific Mass (kg/m ³)	8930	7850	7850

Table 7: Shape properties of model 2

Property Name	Value
Span length (m)	50
Max. distance btw contact & messenger h_0 (m)	0.96
Number of droppers	10

Table 8: Dropper lengths and positions of model 2

Droppers	Longitudinal position of droppers l (m)	Dropper lengths h (m)	
		Cho et al (Cho et al., 2010)	Tur et al (Tur et al., 2014)
1	2.5	0.876	0.8761
2	7.5	0.74	0.7386
3	12.5	0.637	0.6361
4	17.5	0.569	0.5678
5	22.5	0.535	0.5336
6	27.5	0.535	0.5336
7	32.5	0.569	0.5678
8	37.5	0.637	0.6361
9	42.5	0.74	0.7386
10	47.5	0.876	0.8761

BEAM 188 element does not have an option for definition of bending stiffness. Bending stiffness is determined by the program itself with defined young's modulus and sectional properties. However, as mentioned before, messenger cable has a tangential geometry and its bending stiffness is much less than an element having same cross-sectional area with it. Therefore, model of a messenger cable built with BEAM188 does not fit the real bending behavior of it. Although it is not possible to fit the real bending behavior of a cable with BEAM188 as explained above, three models have been built to make a comparison. First one (Model 2-1) has messenger, dropper and contact cables built with LINK10. Second one (Model 2-2) has messenger and dropper cables built with LINK10 and contact cable with BEAM 188. Final one (Model 2-3) has dropper cable built with LINK10, contact and messenger cables with BEAM188 based on its real cross-sectional area. Results of named models built with dropper lengths defined by Cho et al (Cho et al., 2010) and Tur et al (Tur et al., 2014) are shown in Table 9 and Table 10, respectively. Initial tension condition given in Table 6 is not satisfied as in model 1.

Table 9: Results of model of Cho et al (Cho et al., 2010)

Model name	Reaction at contact cable (N)	Reaction at messenger cable (N)
Model 2-1	1432.23	6187.76
Model 2-2	1715.664	6106.577
Model 2-3	1714.065	6106.870

Table 10: Results of model of Tur et al (Tur et al., 2014)

Model name	Reaction at contact cable (N)	Reaction at messenger cable (N)
Model 2-1	1435.432	6188.952
Model 2-2	1717.766	6107.671
Model 2-3	1716.167	6107.964

It is seen that difference between Model 2-1 and Model 2-2 is very high as was in Model 1. This difference is due to the bending stiffness effect of contact cable. In contrast, there is not much difference between Model 2-2 and Model 2-3. This small difference is due to the bending stiffness effect of messenger cable. Compared to contact cable, bending stiffness effect of messenger cable is very small even the cable is modelled with its actual cross-sectional area which ends up with a bending stiffness larger than its actual value.

2.4. Sample Model 2 with Pantograph

This model has same overhead catenary system properties with model 2. Additionally, there is a contact mechanism to model the pantograph. CONTA175 and TARGE169 is used for contact. Pantograph is modelled as a node and displacements are applied to that node. Only dropper lengths proposed by Tur et al (Tur et al., 2014) is used in this model and only contact cable is modelled by BEAM188, other cables are modelled by LINK10. Pantograph is placed at same longitudinal position same with fifth dropper and displacements are applied only in y direction beginning from $y=-0.1$. Resultant reactions at cables' supports and at pantograph for different applied displacements are shown in Table 11. Initial tension condition given in Table 5 is satisfied by applied thermal change.

Table 11: Results of model of Tur et al (Tur et al., 2014)

Applied displacement δ_y (m)	Final position of pantograph y (m)	Reaction at contact cable (N)	Reaction at messenger cable (N)	Reaction at pantograph (N)
0.0	-0.1	12000.000	12000.000	0.000
0.1	0.0	12000.000	12000.000	0.000
0.11	0.01	12000.001	11924.763	17.358
0.12	0.02	12003.002	11828.558	39.906
0.13	0.03	12009.005	11734.358	62.23
0.14	0.04	12016.013	11643.161	84.345
0.15	0.05	12026.018	11556.980	105.38
0.16	0.06	12038.026	11485.827	123.36
0.17	0.07	12053.034	11415.676	141.28
0.18	0.08	12070.035	11346.528	159.17
0.19	0.09	12089.056	11279.382	177.01
0.2	0.1	12111.063	11212.239	194.83

2.5. Sample Model 3 with Pantograph

This model was used in three researches (Lopez-Garcia et al., 2006, Lopez-Garcia et al., 2008, Manabe and Fujii, 1989) [8, 9, and 22]. Inputs are given in Tables 12, 13 and 14. Model is built using LINK10 for messenger cable and droppers and BEAM188 for contact cable as done in previous model. Pantograph is placed at same longitudinal position same with third dropper and displacements are applied

only in y direction beginning from $y=-0.1$. Same contact elements with previous model are used. Reactions versus displacements of pantograph are given in Table 15.

Table 12: Cable properties of model 3

Property Name	Contact Cable	Messenger Cable	Droppers
Mass per Unit Length (kg/m)	0.988	0.697	0.1
Clamp Mass (kg)	0.0	0.0	
Tension (kN)	9.8	9.8	
Elastic Modulus (GPa)	120	200	200
Cross-Sectional Area (mm ²)	110	89	13
Specific Mass (kg/m ³)	8930	7850	7850

Table 13: Shape properties of model 3

Property Name	Value
Span length (m)	50
Max. distance btw contact & messenger h_0 (m)	1.4
Number of droppers	10

Table 14: Dropper lengths and positions of model 3

Droppers	Longitudinal position of droppers l (m)	Dropper lengths h (m)
1	2.5	1.2822
2	7.5	1.0852
3	12.5	0.9375
4	17.5	0.8391
5	22.5	0.7899
6	27.5	0.7899
7	32.5	0.8391
8	37.5	0.9375
9	42.5	1.0852
10	47.5	1.2822

Table 15: Results of model 3

Applied displacement δ_y (m)	Final position of pantograph y (m)	Reaction at pantograph (N)
0.0	-0.1	0.000
0.1	0.0	0.000
0.11	0.01	0.000
0.12	0.02	0.000
0.13	0.03	0.000
0.14	0.04	22.357
0.15	0.05	48.270
0.16	0.06	74.270
0.17	0.07	100.26
0.18	0.08	123.05
0.19	0.09	141.56
0.2	0.1	161.09
0.21	0.11	180.52
0.22	0.12	200.00
0.23	0.13	219.36
0.24	0.14	238.97
0.25	0.15	257.75

A parametric study has been carried out by Lopez-Garcia et al (Lopez-Garcia et al., 2008) to show the effect of pantograph. In that study, pantograph was modelled as a force and displacements through the overhead catenary versus applied force graphs were given. Ones interested in can compare the results of this paper (Table 15) with results given as graph by Lopez-Garcia et al (Lopez-Garcia et al., 2008).

3. Results and Discussion

Static analysis of overhead catenary has done by a commercial computer program ANSYS. There are many finite elements in use in this program. LINK10 and LINK180 are the ones used for cable modelling. However, these finite elements do not have bending stiffness property. Therefore, BEAM188 was used to model the cable considering its bending property. Mainly, three models have been solved by using these elements to see the changes element by element or model by model. Results were tabulated under their title.

The difference between LINK10 and LINK180 was shown by model 1. It can be seen in Table 4 that; there is almost no difference between LINK10 and LINK180. LINK10 has an advantage of convergence due to its additional stiffness option. Therefore, LINK10 was used for other models. Bending stiffness effect of contact cable has been shown by modeling the contact cable either with LINK10 and BEAM188. Difference can be seen by comparing Table 4 and 5. Model 2 was built up with three different finite element combinations to see the bending stiffness effect of both contact cable and messenger cable. In both Table 9 and 10, it can be seen that there is much difference between results of Model 2-1 and 2-2 which reflects the bending stiffness effect of contact cable. In contrast, it can be seen in both table that, there is not much difference between results of Model 2-2 and 2-3 which reflects the bending stiffness effect of messenger cable. Therefore, messenger cable was modelled by LINK10 in following models.

A pantograph was modelled as a contact point and displacements were applied in model 2. Changes in reactions at cables' supports and pantograph with increasing applied displacements to pantograph are shown in Table 11. Besides, pantograph effect was searched for another model.

4. Conclusions

In brief, overhead catenary was analyzed statically by using different finite elements in ANSYS. It was desired to see the bending stiffness effect of cable element and the pantograph effect. In conclusion, it was seen that there is no difference between finite elements LINK10 and LINK180. Besides, bending stiffness effect of contact cable is much greater than of messenger cable. Finally, pantograph effect was shown by models built with messenger and dropper cables by LINK10 and contact cable by BEAM188.

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